accelerates, passes the minimum in ϕ, comes to rest at $y\leq\frac{3}{2}$, then returns toward its initial position, which it fails to reach because of dissipative forces, however small they may be. It then oscillates forever, unless it is overdamped, eventually coming to rest at $y=1$ to satisfy the boundary condition of Eq. (22) as $\xi\rightarrow\infty$.

As the amplitude of oscillation decays, the frequency of oscillation increases and the center of oscillation shifts toward larger values of y, i.e., toward $y=1$. Both these effects are consequences of the anharmonic potential of Eq. (24). The dependence of frequency on amplitude can be estimated from a perturbation calculation. Set $\eta=0$ in Eq. (23) and let $y=x+1$. Then Eq. (23) becomes

$$x''+x+x^2=0, \tag{25}$$

where $x'=dx/d\tau$, etc. Now if we let

$$x=x^{(1)}+x^{(2)}+x^{(3)}+\cdots$$
$$\omega=1+\omega^{(1)}+\omega^{(2)}+\cdots, \tag{26}$$

where successive terms are decreasing in magnitude, we find that[8]

$$x=-a^2/2+a\cos\omega\tau+(a^2/6)\cos2\omega\tau+(a^3/48)\cos3\omega\tau+\cdots \tag{27}$$

$$\omega=1-5a^2/12+\cdots. \tag{28}$$

For comparison, we have integrated Eq. (25) numerically for three values of a with the results shown in Table I. The two sets of results are comparable but far from identical. If a is corrected to yield the correct values of $x(0)=-0.25$, -0.50, -0.75, from Eq. (27), the agreement is somewhat improved. In any event the analysis confirms two points suggested by Fig. 4: The frequency of oscillation decreases and the center of oscillation moves to the left as the amplitude increases.

Equation (12) has been integrated numerically with the boundary condition $S_1'=u_1=$constant for several values of α and u_1 with $\eta=0$. The variation of u_N with time has been determined for $N=30$, 60, and 90, and from each of these functions, which have the form shown in Fig. 1, the amplitude and period of oscillation have been determined as functions of time. Pertinent data for four integrations are given in Table II. Period, in units of ξ, is shown in Fig. 5 as a function of amplitude and $u_1\alpha$ for fixed N. The increase in amplitude with N and the dependence of period on N for fixed $u_1\alpha$ are shown in Fig. 6.

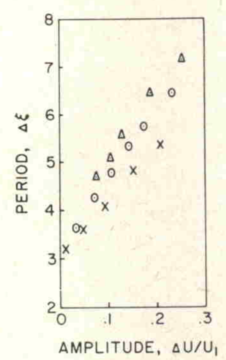

FIG. 6. Effects of travel distance (N) on amplitude and period: $\alpha=0.3$, $u_1=0.1$: **x**—$N=30$; ⊙—$N=60$; △—$N=90$.

[8] L. D. Landau and E. M. Lifshitz, *Mechanics* (Pergamon Press, Inc., New York, 1960), Vol. 1 of Course of Theoretical Physics, p. 86.

V. DISCUSSION AND CONCLUSIONS

Numerical results from the transient problem [Eq. (12)] show that the peak amplitude of oscillation of the Nth particle increases with N for fixed α and with α for fixed N. The former result suggests that when N is very large, $u_{\min}/u_1\rightarrow0$, in accord with Eq. (23) and Fig. 4 and with the concept that the permanent regime solution for the lattice is a steady oscillation in which u returns periodically to zero. The prediction of Fig. 4 that the maximum value of u is $\frac{3}{2}u_1$ depends on truncation of the series in Eq. (16) and does not agree with the numerical integration. The increase of amplitude with α for fixed N is in accord with the continuum result that the rate of approach of a shock wave to its permanent regime profile increases with the curvature of the adiabat.

The variation of period with $(\theta/u_1\alpha)^{1/2}$ bears little relation to the result of Eq. (28) and Table I. The zero point period of $\omega=1$ corresponds, in units of ξ, to

$$\Delta\xi=2\pi(\theta/12u_1\alpha)^{1/2}.$$

The coefficient of 2π, shown in Table II, varies more than tenfold for the cases displayed in Fig. 5. Yet the value of $\Delta\xi$ shown there does not vary more than about 60% at $(u-u_1)/u_1\simeq0$, where Eq. (28) has greatest validity. In the next higher approximation to Eq. (16), the period for zero amplitude oscillations is

$$\Delta\xi=2\pi(\theta/12u_1\alpha)^{1/2}(1-2\alpha u_1\theta)^{1/2},$$

which varies even more rapidly with αu_1 than does the previous approximation. This result does indicate, however, that the variation of period with $u_1\alpha$ is sensitive to the truncation of Eq. (16); resolution of this point may depend upon exact solution of Eq. (15). The weight of the evidence presented here is that Eq. (12) is rather a bad approximation to Eq. (15) when $\eta=0$, though it does produce a profile without a discontinuity.

A rather remarkable suggestion which comes from numerical integration of Eq. (12) is that the solution is essentially independent of u_1 and α for fixed value of $u_1\alpha$. The individual factors were varied by a factor of ten while the product was held constant, yet values of u/u_1 in the solution differed by little more than numerical error.